

THE ACOUSTIC WATTMETER, AN INSTRUMENT
FOR MEASURING SOUND ENERGY FLOW

By
C. W. CLAPP

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR
OF PHILOSOPHY IN THE UNIVERSITY OF MICHIGAN



The Acoustic Wattmeter, an Instrument for Measuring Sound Energy Flow

C. W. CLAPP AND F. A. FIRESTONE
University of Michigan, Ann Arbor, Michigan
(Received June 8, 1941)

An acoustic wattmeter has been constructed to measure sound energy flow. It consists of a crystal pressure microphone and a miniature ribbon velocity microphone mounted close together and connected through separate amplifiers and phase equalizing networks to a thermocouple-type audiofrequency wattmeter. Provision is made to measure sound pressure and velocity separately as well as the energy flow represented by their product. The normal acoustic mobility and absorption coefficient of a surface may be computed from measurements of sound energy flow into the surface and sound energy density at the surface. Measurements were made on samples of hairfelt and Type B Acousti-Celotex at normal incidence with the material mounted at the end of a tube, and at random incidence with the material on the floor of a large sound chamber. Referring all results to random incidence, the values of the absorption coefficients obtained by the two methods agree well with each other and are in fair agreement with results obtained by the reverberation-time method.

INTRODUCTION

THE following definition was proposed by the American Standards Association in 1936: "The sound intensity of a sound field in a specified direction at a point is the sound energy transmitted per unit of time in the specified direction through a unit area normal to this direction at the point. The unit is the erg per second per square centimeter but sound intensity may also be expressed in watts per square centimeter."

Sound intensity has been made the basis of the measurement of loudness level and the term is used frequently in discussing sound fields. However, in spite of the fundamental position this quantity has been given in regard to sound phenomena, insofar as the authors are aware, no attempt has yet been made to measure it directly.* The sound engineer is thus in the same position as the electrical engineer who has available voltmeters and ammeters but no wattmeters. The work reported here was undertaken with the object of constructing an instrument to measure sound intensity and of exploring some of its applications to the study of sound fields.

While the definition quoted above gives a perfectly definite meaning to the word "intensity" as applied to sound fields, it is found

that in less formal discussions the word is often used incorrectly, the most common misuse being to associate it with sound pressure level, or potential energy density. To avoid this confusion we propose that henceforth the intensity vector be called the "power vector," and the terms "power density" or simply "energy flow" be used to describe the component of that vector in a specified direction.

It may be shown that power density (W) is given by the equation

$$W_a = \frac{1}{T} \int_0^T p v_a dt, \quad (1)$$

where p is the instantaneous sound pressure and v_a is the instantaneous particle velocity in the direction a in which the power density is being measured. Both pressure and velocity are assumed to be periodic with the common period T . It is clear then that if we can obtain an alternating electric voltage proportional to the instantaneous sound pressure at a point, and another voltage proportional to the component in the a direction of the instantaneous particle velocity at the same point, and apply these two voltages to the terminals of a suitable wattmeter, the reading of the wattmeter will be proportional to the power density W_a at the point. This is the principle used in the instrument described here.

* The principle of an instrument for measuring sound intensity was described in U. S. Patent No. 1892644 issued to Harry F. Olson.

CONSTRUCTION

In practice, the instrument is set up as shown in the block diagram Fig. 1. A crystal pressure microphone and ribbon velocity microphone are mounted together in a single microphone unit. Leads come from these two microphones to separate amplifiers and attenuators. These in turn feed through phase correcting networks to the wattmeter unit. The purpose of the phase correctors is to compensate for phase shift in the microphones or preceding amplifiers.

The wattmeter used here is of the thermocouple type. It was selected because of its stability and the ease with which it could be adapted to work over an extended range of frequencies. The theory of this type of wattmeter has been discussed by Bruckmann and others.¹⁻⁴

Figure 2 shows a simplified version of the actual circuit arrangement in which T_1 and T_2 are two ideal transformers having, for simplicity, a unity turns ratio between the primary and each secondary, and H_1 , H_2 are two matched thermocouples with an ideal square-law output. G is a galvanometer for measuring the d.c. output of the thermocouples connected in series with opposing polarity. Let the voltages applied across the primaries of the two transformers be

$$\begin{aligned} e_1 &= E_1 \sin \omega t, \\ e_2 &= E_2 \sin (\omega t + \theta). \end{aligned}$$

The voltages appearing across the heaters of the thermocouples are then given by

$$e_3 = e_2 + e_1 = E_2 \sin (\omega t + \theta) + E_1 \sin \omega t, \quad (2)$$

$$e_4 = e_2 - e_1 = E_2 \sin (\omega t + \theta) - E_1 \sin \omega t \quad (3)$$

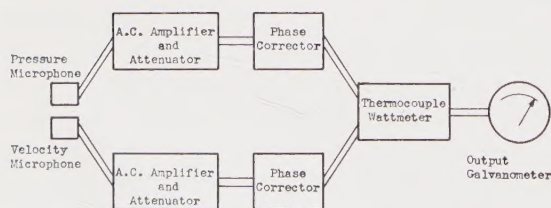


FIG. 1. Block diagram of acoustic wattmeter.

and the d.c. output of the thermocouples by

$$E_5 = k \langle e_3^2 \rangle_{av} = \frac{1}{2} k E_1^2 + \frac{1}{2} k E_2^2 + k E_1 E_2 \cos \theta, \quad (4)$$

$$E_6 = k \langle e_4^2 \rangle_{av} = \frac{1}{2} k E_1^2 + \frac{1}{2} k E_2^2 - k E_1 E_2 \cos \theta, \quad (5)$$

where k is a constant characteristic of the thermocouples, and the angle brackets mean here that the time average is to be taken over one complete period of that quantity. Since the galvanometer measures the difference in the e.m.f.'s of the two thermocouples, its reading will be propor-

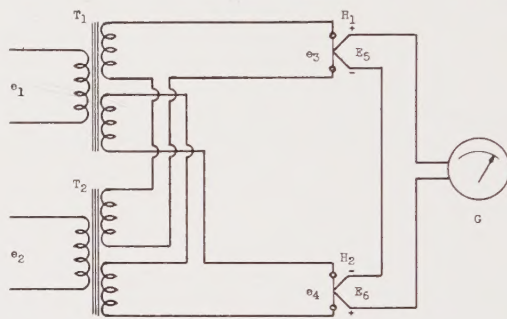


FIG. 2. Simplified circuit of thermocouple wattmeter unit.

tional to the quantity $E_1 E_2 \cos \theta$. The arrangement can therefore be used to measure electrical power in a circuit by making e_1 proportional to the voltage drop across that circuit, and e_2 proportional to the current into it; or it can be used to measure acoustic power by making e_1 and e_2 proportional respectively to the sound pressure and the desired component of particle velocity.

In practice, both the transformers and the thermocouples will depart somewhat from the ideal performance assumed above, but errors from this source may be overcome by careful balancing of the two sides of the wattmeter and by the use of calibration curves.

The complete wattmeter circuit as finally adopted is shown in Fig. 3. The wattmeter unit proper comprises that portion of the circuit to the right of the dashed line, the terminals of the wattmeter being taken at the points $a-b$, $c-d$. Since three stages of amplification are thus included in each side of the wattmeter, this equipment provides a sensitive means of measuring audiofrequency power apart from its special use here. Adjustment of phase shift in the two sides of the circuit was such that the maximum phase angle error in the range from 60 to 2800

¹ W. Bader, Archiv. f. Elektrotechnik 29, 809 (1935).

² H. W. L. Bruckmann and W. J. Reichert, Elektrotech. und Maschinenbau, 48, 781 (1930).

³ A. Herczeg, Elektrotech. Zeits. 51, 421 (1930).

⁴ P. M. Lincoln, J. A. I. E. E. 48, 129 (1929).

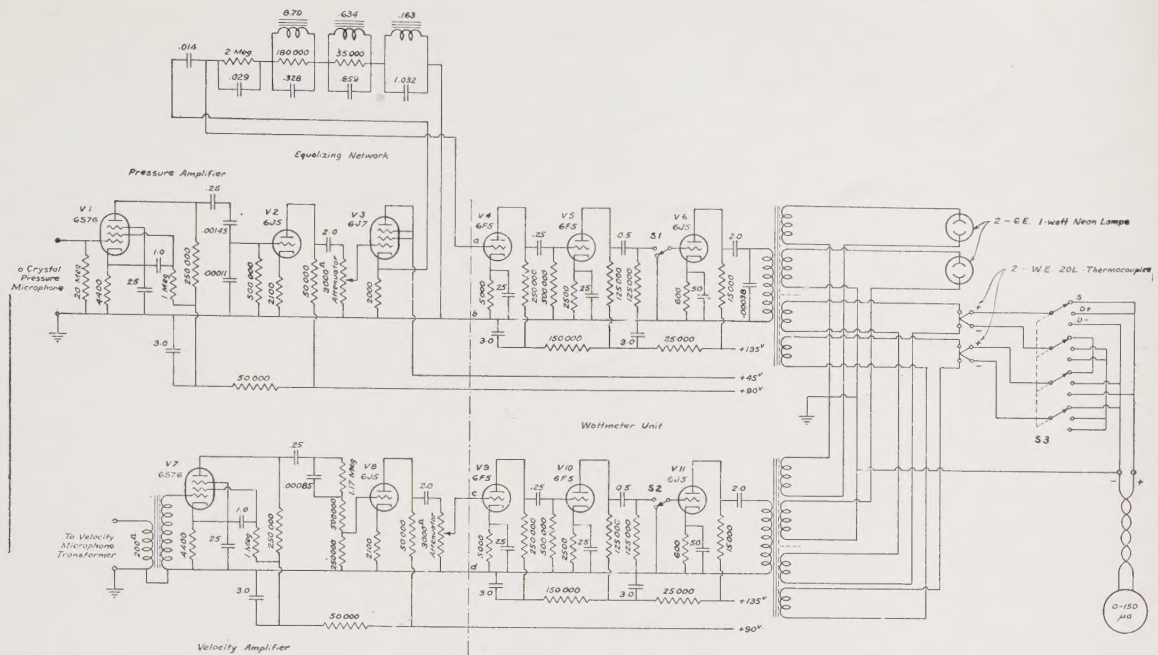


FIG. 3. Wiring diagram of amplifier and wattmeter units.

cycles was 0.6 degree. When compared with a Weston portable wattmeter at 60 cycles, the two agreed within 2 percent of full scale on loads having a power factor of from 100 percent down to 2 percent leading.

When working with the rapidly fluctuating voltages encountered in sound measurements, it is desirable to have some means of protecting the sensitive thermocouples against burnout due to overloads. This was provided here by connecting two 1-watt neon lamps to a set of high impedance secondaries on the output transformers in such a way that the voltages across the lamps were proportional to, but about 25 times greater than the voltages across the thermocouples. The lamps fired when the voltage across either thermocouple was 1.9 volts and this did not rise above 2.0 volts when the input voltage to the wattmeter was raised 40 db. At the same time, the lamps did not affect to any measurable degree the operation of the wattmeter at normal signal level giving a heater voltage of about 0.7 volt.

By means of the switches S_1 , S_2 , S_3 it is possible to connect the wattmeter to measure the mean square value of voltages connected to either of its input terminals. This permits a measurement

of the kinetic and potential sound energy density at the microphone and adds greatly to the usefulness of the instrument as will be seen later.

The attenuators preceding the wattmeter unit were found to give some trouble at first because the phase shift introduced by them varied with the attenuator setting. This effect was reduced to a negligible size by proper choice of the attenuator impedance and of the output impedance of the stage preceding them. Great care must be exercised to prevent coupling between the pressure and the velocity sides of the wattmeter, or between any two circuits on opposite sides of an attenuator. The effect of any such coupling is to make the phase shift in the wattmeter depend on the signal level or attenuator setting.

Figure 4 shows the construction of the microphone unit containing the pressure microphone, velocity microphone, and ribbon-to-line transformer. The pressure microphone was made up from two Standard Brush Sound Cells mounted one at either end of the velocity microphone and connected in parallel. This arrangement makes it possible to have the geometrical center of the pressure microphone coincide with the center of the velocity microphone. The double-horseshoe-

shaped magnetic structure of the velocity microphone was machined in one piece from a bar of cobalt magnet steel and permanently magnetized. The ribbon was cut from aluminum foil rolled down to a thickness of 0.0001" and was corrugated lightly. In over-all dimensions, the velocity microphone measured 2 inches long, $\frac{1}{2}$ inch wide and $\frac{1}{4}$ inch thick. The sensitivity of the pressure microphone at 1000 cycles was 75 decibels below 1 volt/bar, and of the velocity microphone including ribbon-to-line and line-to-grid transformers was approximately 50 decibels below 1 volt/kine.

The operation of the pressure microphone was in general satisfactory. The crystals were free from resonances and diffraction effects up to at least 2000 cycles which was the limit of our means of calibration. In addition, they were compact and convenient to use. However, as was to be expected, the phase shift and sensitivity of the crystals was somewhat dependent on temperature. This became serious at temperatures above 25°C. Fortunately, all the work reported here could be done in the temperature range from 24 to 25.5°C—it was August, 1939—in which the total change in phase at 125 cycles amounted to less than 1.5 degrees and the change in sensitivity was 0.5 decibel. The magnitude of the variations was approximately inversely proportional to the frequency and was not troublesome at higher frequencies.

The velocity microphone presented a difficult problem in connection with its ribbon resonances. While the fundamental mode of vibration can usually be kept well below the working frequency range, almost invariably one or more of the higher modes of vibration is of sufficient importance to cause trouble. This it does by introducing large changes in the phase and amplitude of the output of the microphone for sounds having a frequency close to one of these resonance frequencies, and large errors are introduced in the reading of sound power density unless these changes, and especially the changes in phase, are corrected by suitable networks.

CORRECTION OF RIBBON RESONANCES

The mobility of a simple mechanical element has been defined⁵ as the ratio of the velocity of

(or across) the element to the force applied to (or through) the element. This same concept of mobility may be applied to the more complex mechanical structure presented by a stretched microphone ribbon by making a few simplifying assumptions, *viz.*:

(1) The only components of force and velocity which we need consider are those whose direction is normal to the plane of the ribbon.

(2) The velocity of the ribbon is taken to be the average normal velocity weighted according to some given function of position (h). Thus

$$v_E = -\frac{1}{A} \int_A \frac{\partial y}{\partial t} h(x, z) dx dz, \quad (6)$$

where A is the total area of the ribbon, $\partial y/\partial t$ is the normal velocity of the ribbon element $dx dz$, and v_E is the quantity which we will henceforth call the effective velocity of the ribbon. Later, it will be convenient to associate the weighting

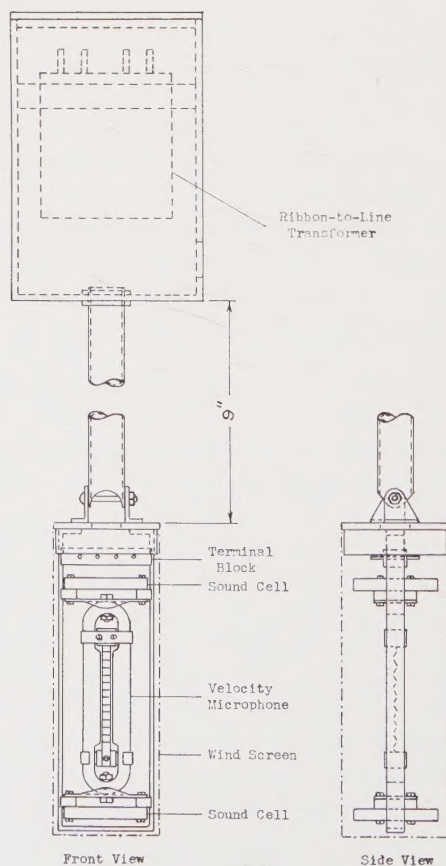


FIG. 4. Construction of microphone unit.

⁵ F. A. Firestone, J. Acous. Soc. Am. 4, 249 (1933).

function $h(x, z)$ with the strength of the transverse magnetic field in which the ribbon lies.

(3) With special reference to the problem at hand, it is also convenient to assume that the force arises from a difference in pressure on the two sides of the ribbon, and that the pressure is uniform over each side. This means that for sound waves striking the ribbon obliquely, the difference in phase along the ribbon must not be appreciable.

With these initial assumptions, let us proceed to find the mobility of a stretched flexible ribbon supported at its two ends and acted upon by a uniformly distributed sinusoidal force. The ribbon may be supposed to be stretched along the X axis of a rectangular coordinate system between the points $x=0$ and $x=l$. Let the mass of the ribbon be m grams per centimeter length, and the tension T dynes. If we assume that the driving force is $fe^{j\omega t}$ dynes per centimeter length acting in the positive Y direction, and that frictional forces amounting to r dynes per kine per centimeter length are also acting, the equation of motion of the ribbon is

$$m \frac{\partial^2 y}{\partial t^2} + r \frac{\partial y}{\partial t} - T \frac{\partial^2 y}{\partial x^2} - fe^{j\omega t} = 0. \quad (7)$$

The boundary conditions are $y=0$ at $x=0$ and $x=l$. The solution of this boundary value problem is found by standard methods to be

$$y = \sum_{n=1, 2}^{\infty} \left(\frac{2}{l} \int_0^l f \sin \frac{n\pi x}{l} dx \right) \times \frac{\sin n\pi x/l}{Tn^2\pi^2/l^2 - \omega^2 m + j\omega r} e^{j\omega t}. \quad (8)$$

Since the driving force f is assumed to be independent of x , the integral is readily evaluated and we get

$$y = \sum_{n=1, 3, 5}^{\infty} \frac{4f/n\pi \sin n\pi x/l}{Tn^2\pi^2/l^2 - \omega^2 m + j\omega r} e^{j\omega t} = \sum_{n=0, 1, 2}^{\infty} \frac{4f/(2n+1)\pi \sin (2n+1)\pi x/l}{T(2n+1)^2\pi^2/l^2 - \omega^2 m + j\omega r} e^{j\omega t}. \quad (9)$$

The effective velocity of the ribbon is given by Eq. (6) and if we now assume that the weighting

function $h(x, z)$ is a constant equal to unity, then

$$v_E = \frac{1}{l} \int_0^l \frac{\partial y}{\partial t} dx = \frac{1}{l} \int_0^l \left[\sum_{n=0, 1, 2}^{\infty} \frac{j4\omega f \sin \frac{(2n+1)\pi x}{l}}{T(2n+1)^2\pi^2 - \omega^2 m + j\omega r} e^{j\omega t} \right] dx = \sum_{n=0, 1, 2}^{\infty} \frac{j8\omega f}{T(2n+1)^2\pi^2 - \omega^2 m + j\omega r} e^{j\omega t}. \quad (10)$$

But the total force acting on the ribbon is equal to $fle^{j\omega t}$ hence the mobility of the ribbon is

$$Z_R = \frac{v_E}{fle^{j\omega t}} = \sum_{n=0, 1, 2}^{\infty} \frac{j8\omega}{T(2n+1)^2\pi^2 - \omega^2 m + j\omega r}. \quad (11)$$

Substituting

$$R_n = \frac{8}{(2n+1)^2\pi^2 l r},$$

$$C_n = \frac{1}{8}(2n+1)^2\pi^2 l m,$$

$$L_n = \frac{8l}{(2n+1)^4\pi^4 T},$$

we obtain the mobility of the ribbon in the form

$$Z_R = \sum_{n=0}^{\infty} Z_n = \sum_{n=0}^{\infty} \frac{1}{\frac{1}{R_n} + j\left(\omega C_n - \frac{1}{\omega L_n}\right)}. \quad (12)$$

Hence the mobility of the ribbon can be thought of as an infinite series of component mobilities Z_n , each of which consists of a dissipative term R_n , a compliance L_n , and a mass C_n connected in parallel. The mobility of the separate elements diminish rapidly as n increases. Note also that because of the assumed uniformity of the magnetic field, only the fundamental and odd modes of vibration of the ribbon contribute to the mobility.

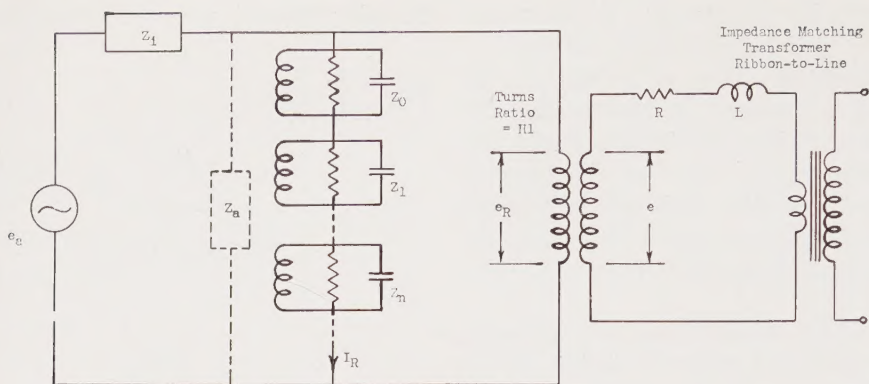


FIG. 5. Equivalent circuit of ribbon velocity microphone.

Figure 5 shows an equivalent circuit of the ribbon velocity microphone. The mobility of the ribbon is represented by the string of parallel resonance circuits $Z_0, Z_1, \dots, Z_n, \dots$. The e.m.f. of the generator e_a is taken to be proportional to the velocity at the position of the microphone in the undisturbed wave, while the mobilities Z_i, Z_a , account for the local distortion of the wave by the microphone structure. The ideal transformer of turns ratio Hl marks the transition from mechanical and acoustical elements to purely electrical elements, R and L being the electrical resistance and inductance of the ribbon and leads.

The equivalent circuit was used as a guide to the design of the equalizing network, the plan being to overcome the effects of phase shift in the velocity microphone by introducing a corresponding phase shift in the pressure side of the wattmeter. The equalizing network was therefore built up having the same arrangement of resonance circuits, etc., as the equivalent circuit of the velocity microphone and was inserted following a special low output impedance stage in the pressure amplifier. (See Fig. 3.)

In Fig. 6 is plotted the phase shift and sensitivity of the velocity microphone and amplifier system referred to that of the pressure microphone and amplifier. The measurements of phase angle and sensitivity shown here were made by setting up a standing wave system in a long brass tube closed by a rigid wall at one end and excited at the other. The microphones were mounted a distance of $\frac{1}{8}$ wave-length from the

reflecting end of the tube. At such a point in the sound field the potential energy density is equal to the kinetic energy density and the pressure and particle velocity are 90° apart in phase. As a check on possible absorption in the tube, measurements were repeated at $\frac{3}{8}$ wave-length from the closed end. The dashed line in Fig. 6 indicates the phase shift caused by the input transformers on the velocity side. Ribbon resonances are clearly indicated at frequencies of approximately 100, 225, 410 and 1150 cycles. Figure 7 shows the same curves taken after the equalizing network was installed. The 100- and 225-cycle resonances have been practically wiped out. Equalization of the 410-cycle resonance was not complete because of its sharpness and the consequent difficulty in obtaining an inductance having a sufficiently high " Q ." The smaller resonance at 1150 cycles was missed in the first calibration of the microphone and was left uncorrected.

THEORY OF MEASUREMENTS WITH THE ACOUSTIC WATTMETER

The absorption coefficient of a surface is a quantity to which a great deal of attention has been paid, and it is of some interest to see how the acoustic wattmeter can be used to measure this quantity. We begin by considering the case in which a plane progressive wave is incident normally on the surface of a uniform layer of the absorbing material. This is the condition generally assumed and approximately realized in a measurement of absorption coefficient by any

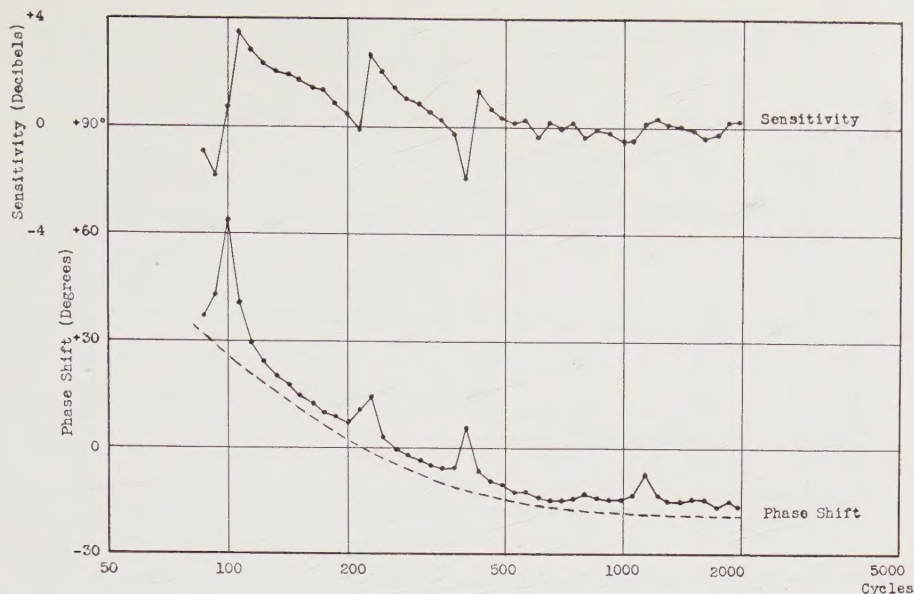


FIG. 6. Phase shift and sensitivity of velocity microphone as a function of frequency.

of the conventional tube methods.⁶⁻⁸ In these, measurements are made of the pressure or velocity in the longitudinal standing wave system which can be set up in a long narrow tube, one end of which is closed by the absorbing material under test. The velocity potential for such a field can be written

$$\phi = A \cosh(u - jkx), \quad (13)$$

where $A = 2(A_1 A_2)^{1/2}$,

$$u = \log(A_1/A_2)^{1/2},$$

$$k = 2\pi/\lambda = \omega/c,$$

x = distance measured from equivalent reflecting plane,

A_1 = amplitude of wave incident on absorbing material,

A_2 = amplitude of wave reflected from absorbing material.

The sound pressure and particle velocity in the tube are given by

$$p = j\omega\rho_0 A \cosh(u - jkx), \quad (14)$$

$$v = jkA \sinh(u - jkx), \quad (15)$$

where ρ_0 is the density of the air in the tube. From these we can compute that the sound power density in the direction of the tube axis, averaged over one complete period of the wave is

$$W_x = \frac{1}{2}\omega\rho_0 k A^2 \sinh u \cosh u \\ = \frac{1}{2}\omega\rho_0 k (A_1^2 - A_2^2). \quad (16)$$

The sound energy density, also averaged over one complete period is

$$E = \frac{1}{4}\rho_0 k^2 A^2 (\cosh^2 u + \sinh^2 u) \\ = \frac{1}{2}\rho_0 k^2 (A_1^2 + A_2^2) \quad (17)$$

and the specific acoustic mobility (Z_x) looking in the direction of the absorbing material is given by

$$\rho_0 c Z_x = \tanh(u - jkx). \quad (18)$$

The energy absorption coefficient α is given in terms of the amplitudes of the direct and reflected waves by the relation

$$\alpha = 1 - \frac{|A_2|^2}{|A_1|^2}.$$

From Eqs. (16) and (17) above, we see that

$$\frac{W_x}{cE} = \frac{|A_1|^2 - |A_2|^2}{|A_1|^2 + |A_2|^2}.$$

⁶ E. T. Paris, Proc. Phys. Soc. **39**, 269 (1927).

⁷ E. C. Wentz and E. H. Bedell, Bell Sys. Tech. J. **7**, 1 (1928).

⁸ A. H. Davis and E. J. Evans, Proc. Roy. Soc. **127**, 89 (1930).

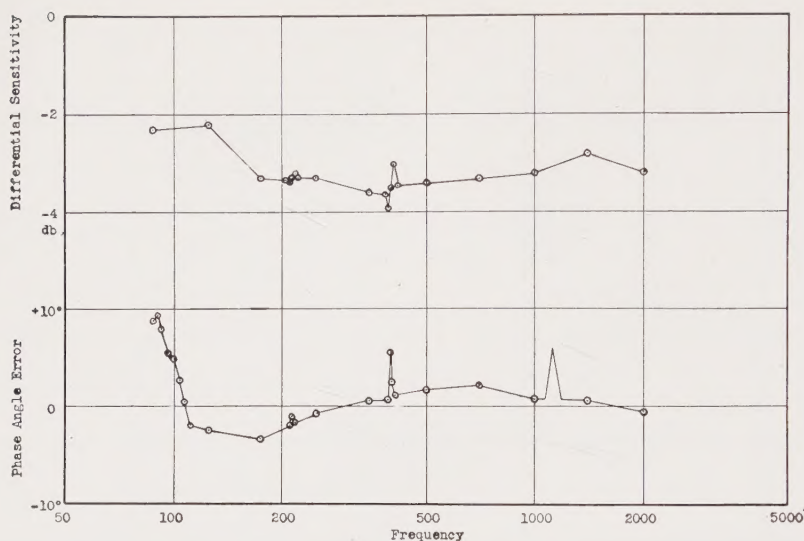


FIG. 7. Phase angle error and differential sensitivity of acoustic wattmeter as a function of frequency.

Therefore

$$\alpha_0 = \frac{2(W_x/cE)}{1 + (W_x/cE)}. \quad (19)$$

The subscript 0 is used with α here to indicate that it is the coefficient for waves at normal incidence. Since both the power density (W_x) and the energy density (E) are independent of x ,

the position along the tube, Eq. (19) enables us to compute the absorption coefficient from two measurements which may be made with the acoustic wattmeter at any point in the tube.

The absorption coefficient may also be found from measurements of the acoustic mobility of the tube. By Eq. (18), a measurement of the real and imaginary components of $\rho_0 c Z_x$ is sufficient to determine the quantity u , in terms of which the absorption coefficient is given by the relation

$$\alpha_0 = 1 - e^{-4u}. \quad (20)$$

Here again, the measurements from which α_0 is computed may be made at any point in the tube.

The absorption coefficient of a given material usually varies with the angle of incidence of the sound wave striking it. For this reason, measurements of the normal incidence coefficient have only limited application for materials to be used under conditions of random incidence unless suitable corrections can be applied to the values obtained at normal incidence. The more usual procedure is to find an average coefficient directly by observing, through measurements of reverberation times, the influence of the sample on a supposedly random sound field in which waves are striking the sample from all directions. The following is an attempt to develop a similar technique for use with the acoustic wattmeter.

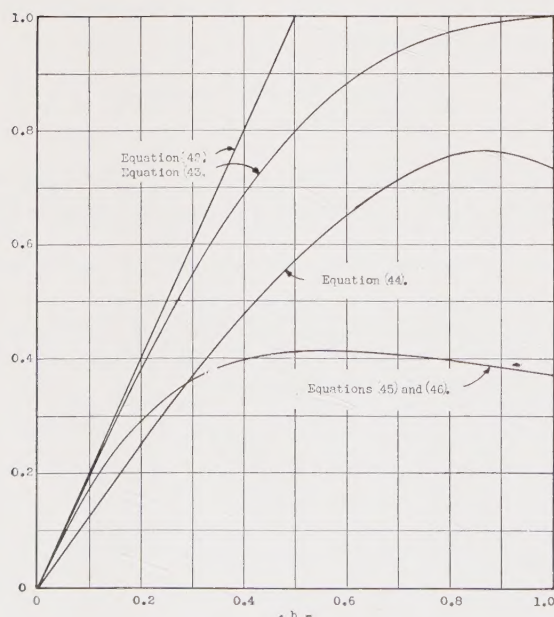


FIG. 8. Plot of right-hand members of Eqs. (42) to (46) as functions of b .

We begin with the simple case of a plane progressive wave incident at angle θ on a plane reflecting surface, and we make the assumption that the mobility of the reflecting surface is a pure real quantity which is independent of θ . If we choose a rectangular coordinate system in which the X axis is normal to the reflecting surface and the Y axis lies in the plane defined by the X axis and the direction of propagation of the wave, then the velocity potential for the incident wave can be written

$$\phi_1 = A_1 \exp [j(\omega t - kx \cos \theta + ky \sin \theta)] \quad (21)$$

and for the reflected wave,

$$\phi_2 = A_1 r \exp [j(\omega t + kx \cos \theta + ky \sin \theta)], \quad (22)$$

where A_1 = amplitude of the incident wave, which can be taken as real without any loss in generality.

r = reflection coefficient, which is real since the mobility of the surface is assumed real.

Writing $\xi = x \cos \theta$, $\eta = y \sin \theta$, and suppressing the time varying factor, the velocity potential for the sound field in front of the reflecting plane can therefore be written

$$\begin{aligned} \phi &= \phi_1 + \phi_2 \\ &= A_1 [(1+r) \cos k\xi \cos k\eta \\ &\quad + (1+r) \sin k\xi \sin k\eta] \\ &\quad + jA_1 [(1+r) \cos k\xi \sin k\eta \\ &\quad - (1-r) \sin k\xi \cos k\eta]. \end{aligned} \quad (23)$$

$$Z_n = \frac{v_x}{p} = \frac{\cos \theta}{\rho_0 c} \frac{[(1+r) \sin k\xi \cos k\eta - (1-r) \cos k\xi \sin k\eta]}{[(1-r) \sin k\xi \cos k\eta - (1+r) \cos k\xi \sin k\eta]}$$

The potential energy density is

$$U = \frac{p^2}{2\rho_0 c^2} = \frac{1}{4} \rho_0 k^2 A_1^2 [(1+r)^2 - 4r \sin^2 k\xi] \quad (29)$$

and the kinetic energy densities associated with the normal (X) and parallel (Y) components of velocity are

From this we can compute the sound pressure

$$\begin{aligned} p &= \rho_0 \frac{\partial \phi}{\partial t} \\ &= \omega \rho_0 A_1 [(1-r) \sin k\xi \cos k\eta \\ &\quad - (1+r) \cos k\xi \sin k\eta] \\ &\quad + j\omega \rho_0 A_1 [(1+r) \cos k\xi \cos k\eta \\ &\quad + (1-r) \sin k\xi \sin k\eta] \end{aligned} \quad (24)$$

and the velocity components in the X and Y directions

$$\begin{aligned} v_x &= -\frac{\partial \phi}{\partial x} = -\cos \theta \frac{\partial \phi}{\partial \xi} \\ &= kA_1 \cos \theta [(1+r) \sin k\xi \cos k\eta \\ &\quad - (1-r) \cos k\xi \sin k\eta] \\ &\quad + jkA_1 \cos \theta [(1+r) \sin k\xi \sin k\eta \\ &\quad + (1-r) \cos k\xi \cos k\eta], \end{aligned} \quad (25)$$

$$\begin{aligned} v_y &= -\frac{\partial \phi}{\partial y} = -\sin \theta \frac{\partial \phi}{\partial \eta} \\ &= kA_1 \sin \theta [(1+r) \cos k\xi \sin k\eta \\ &\quad - (1-r) \sin k\xi \cos k\eta] \\ &\quad - jkA_1 \sin \theta [(1+r) \cos k\xi \cos k\eta \\ &\quad + (1-r) \sin k\xi \sin k\eta]. \end{aligned} \quad (26)$$

The power density in the positive X direction (the normal into the reflecting surface) is given by the scalar product of p and v_x considered as vectors in phase space, thus

$$W_n = p \cdot v_x = \frac{1}{2} \omega \rho_0 k A_1^2 (1-r^2) \cos \theta. \quad (27)$$

The mobility in the same direction is

$$\times \frac{+j(1+r) \sin k\xi \sin k\eta + j(1-r) \cos k\xi \cos k\eta}{+j(1+r) \cos k\xi \cos k\eta + j(1-r) \sin k\xi \sin k\eta}. \quad (28)$$

$$\begin{aligned} T_n &= \frac{1}{2} \rho_0 v_x^2 \\ &= \frac{1}{4} \rho_0 k^2 A_1^2 \cos^2 \theta [(1-r)^2 + 4r \sin^2 k\xi], \end{aligned} \quad (30)$$

$$\begin{aligned} T_p &= \frac{1}{2} \rho_0 v_y^2 \\ &= \frac{1}{4} \rho_0 k^2 A_1^2 \sin^2 \theta [(1+r)^2 - 4r \sin^2 k\xi]. \end{aligned} \quad (31)$$

Returning to Eq. (28), the mobility of the reflecting surface itself is obtained by putting $x=0$.

Hence

$$(Z_n)_{x=0} = \frac{\cos \theta (1-r)}{\rho_0 c (1+r)}. \quad (32)$$

Or, writing $b = \rho_0 c (Z_n)_{x=0}$, we obtain the familiar equation

$$r = \frac{\cos \theta - b}{\cos \theta + b}, \quad (33)$$

which gives the relation between reflection coefficient and angle of incidence when the normal mobility of the reflecting surface is real and independent of θ .

To provide a basis for the measurement of the absorption coefficient in a closed room by means of the acoustic wattmeter, we now make the assumption that the above simple theory is adequate to describe the sound field in the neighborhood of an absorbing boundary of the room. Some justification for this is found in the exact solution for a room whose shape is that of a rectangular parallelepiped having one wall completely covered by an absorbing material, the four adjacent walls by perfectly reflecting materials, and the opposite wall by a surface distribution of sound sources. The analysis of the steady-state sound field in such a space yields results differing only in minor respects with those obtained in the last section. For instance, if we choose the Y and Z axes of our coordinate system in the plane of the absorbing wall, and if we average the quantities W_n , U , T_n , and T_p ($=T_y+T_z$) separately over y and z within the limits of the room, we obtain a set of quantities whose variation with x is the same as that given by Eqs. (27), (29), (30) and (31).

From Eq. (27) we find that at the absorbing boundary (or indeed for any value of x)

$$W_n = \frac{1}{2} \omega \rho_0 k A_1^2 (1-r^2) \cos \theta.$$

Inserting the value of r in terms of θ and b from Eq. (33)

$$W_n = 2 \omega \rho_0 k A_1^2 b \cos^2 \theta / (\cos \theta + b)^2. \quad (34)$$

Now let us suppose that the actual sound field is made up, not of one single wave or mode of vibration, but of many such waves or modes, so that all directions in space are equally represented. If each of the incident waves in this "random" sound field can be supposed to have

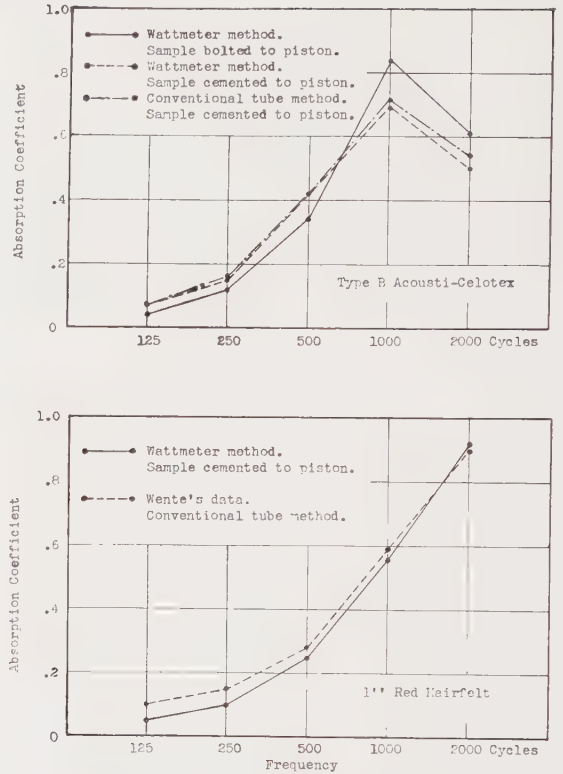


FIG. 9. Absorption coefficients measured at normal incidence.

the same power density, i.e., $\frac{1}{2} \omega \rho_0 k A_1^2 = \text{constant}$, then the average power density normal to the boundary is found by averaging the expression for W_n above for waves arriving from all directions. We have therefore

$$\begin{aligned} \bar{W}_n &= 2 \omega \rho_0 k A_1^2 b \int_0^{\pi/2} \frac{\cos^2 \theta \sin \theta d\theta}{(\cos \theta + b)^2} \\ &= 2 \omega \rho_0 k A_1^2 b \left[\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right], \quad (35) \end{aligned}$$

where the bar over a quantity is used here to designate an average taken over all possible directions of the incident wave.

Similarly, inserting $x=0$ in Eqs. (29), (30), (31) and designating the energy densities at the surface of the absorbing material by the subscript 0, we have

$$\begin{aligned} U_0 &= \frac{1}{4} \rho_0 k^2 A_1^2 (1+r)^2, \\ T_{n0} &= \frac{1}{4} \rho_0 k^2 A_1^2 (1-r)^2 \cos^2 \theta, \\ T_{p0} &= \frac{1}{4} \rho_0 k^2 A_1^2 (1+r)^2 \sin^2 \theta. \end{aligned}$$

Inserting the value of r as a function of θ and

averaging over all directions,

$$\bar{U}_0 = \rho_0 k^2 A_1^2 \left[\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right], \quad (36)$$

$$\bar{T}_{n0} = \rho_0 k^2 A_1^2 b^2 \left[\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right], \quad (37)$$

$$\bar{T}_{p0} = \rho_0 k^2 A_1^2 \left[(1-2b^2) \times \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right) + \frac{2b-1}{3(b+1)} \right]. \quad (38)$$

If, instead of making the energy density measurements at the surface of the absorbing material only, we take a sufficient number of readings at points distributed throughout the whole volume of the chamber to ensure an average value for the whole room, then designating such space averages by the subscript s , we have

$$\begin{aligned} U_s &= \frac{1}{4} \rho_0 k^2 A_1^2 (1+r^2), \\ T_{ns} &= \frac{1}{4} \rho_0 k^2 A_1^2 (1+r^2) \cos^2 \theta, \\ T_{ps} &= \frac{1}{4} \rho_0 k^2 A_1^2 (1+r^2) \sin^2 \theta. \end{aligned}$$

Again inserting the value of r as a function of θ and averaging over all directions,

$$\bar{U}_s = \frac{1}{2} \rho_0 k^2 A_1^2 \left[\frac{3b+1}{b+1} - 2b \log \frac{b+1}{b} \right], \quad (39)$$

$$\bar{T}_{ns} = \frac{1}{2} \rho_0 k^2 A_1^2 \left[3b^2 \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right) - \frac{2b-1}{3(b+1)} \right], \quad (40)$$

$$\begin{aligned} \bar{T}_{ps} &= \frac{1}{2} \rho_0 k^2 A_1^2 \left[(1-3b^2) \right. \\ &\quad \times \left. \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right) + \frac{5b-1}{3(b+1)} \right]. \quad (41) \end{aligned}$$

By various combinations of the above we obtain the following ratios as being the most useful for a determination of b and hence of the absorption coefficient.

$$\bar{W}_n / c \bar{U}_0 = 2b, \quad (42)$$

$$\bar{W}_n / c (\bar{U}_0 + \bar{T}_{n0}) = 2b / (1+b^2), \quad (43)$$

$$\frac{\bar{W}_n}{c(\bar{U}_0 + \bar{T}_{n0} + \bar{T}_{p0})} = \frac{2b \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right)}{(2-b^2) \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right) + \frac{2b-1}{3(b+1)}}, \quad (44)$$

$$\frac{\bar{W}_n}{2c \bar{U}_s} = \frac{2b \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right)}{\frac{3b+1}{b+1} - 2b \log \frac{b+1}{b}}, \quad (45)$$

$$\frac{\bar{W}_n}{2c(\bar{T}_{ns} + \bar{T}_{ps})} = \frac{2b \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right)}{\frac{3b+1}{b+1} - 2b \log \frac{b+1}{b}}. \quad (46)$$

The next step is to find the "average" absorption coefficient corresponding to any given value of b . The absorption coefficient is defined as the ratio of the energy absorbed by a given surface

in a given time to the total energy incident on that surface in the same time. We therefore define the average or random-incidence absorption coefficient as the ratio of the absorbed

energy to the incident energy when waves of equal energy density are reaching the absorbing surface from all directions in space. Referring again to Eq. (17), the total energy density in the incident wave alone is found to be

$$E = \frac{1}{2} \rho_0 k^2 A_1^2 \quad (47)$$

and the energy per second striking unit area of surface inclined at angle θ to the direction of the wave is

$$cE \cos \theta = \frac{1}{2} \omega \rho_0 k A_1^2 \cos \theta. \quad (48)$$

For waves of equal strength arriving at the surface from all directions, the average power reaching unit area of the surface is therefore

$$\begin{aligned} cE \langle \cos \theta \rangle_{av} &= \frac{1}{2} \omega \rho_0 k A_1^2 \int_0^{\pi/2} \cos \theta \sin \theta d\theta \\ &= \frac{1}{4} \omega \rho_0 k A_1^2. \end{aligned} \quad (49)$$

The rate of energy absorption under the same conditions is given by Eq. (35), hence dividing Eq. (35) by Eq. (49), we obtain the random incidence absorption coefficient

$$\bar{\alpha} = 8b \left(\frac{2b+1}{b+1} - 2b \log \frac{b+1}{b} \right). \quad (50)$$

MEASUREMENTS WITH THE ACOUSTIC WATTMETER

In Fig. 9 is shown the results of some of the measurements of absorption coefficient at normal incidence using the acoustic wattmeter. To obtain the data for these curves, standing waves of sound were set up in a long brass tube 5 inches in diameter. One end of the tube was closed by a piston whose face was covered by the absorbing material under test. The other end was closed by a bare metal piston having a small hole at its center through which the sound waves entered. The microphone unit of the wattmeter was mounted in the tube facing the sample and about $\frac{3}{8}$ wave-length away. Measurements were made of sound energy density and power density and the normal absorption coefficient computed using Eq. (19). In one case—the curve marked “conventional tube method”—the absorption coefficient was computed from measurements of the sound pressure at the nodes and antinodes

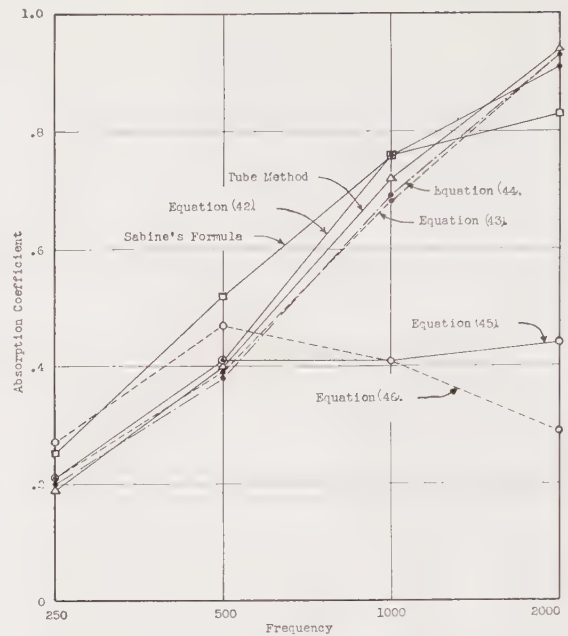


FIG. 10. Random incidence absorption coefficients for hairfelt.

of the standing wave system. With Type B Acousti-Celotex, the difference between the results by the wattmeter method and by the “tube” method was less than the variation due to the method of mounting the sample. In the lower set of curves, the dashed line is taken from Wentz and Bedell’s data⁷ obtained by a tube method. The agreement is considered good in view of the variability of this material.

Measurements were also made on large scale pieces of the same materials. The samples, approximately 6 feet by 14 feet in size, were laid on the floor of the reverberation room at the University of Michigan (4620 cu. ft.) and a loudspeaker source was mounted along one side wall. Although these conditions do not conform exactly to the conditions assumed in obtaining these equations, it was of some interest to see to what extent they would still be valid. To provide some degree of randomness in the sound field, the frequency of the source was varied through a band spreading about 20 percent on either side of the mean frequency at a rate of about 2 warbles per second. At nine different positions on the sample, readings were taken of the normal component of power density, the potential energy density, and the kinetic energy

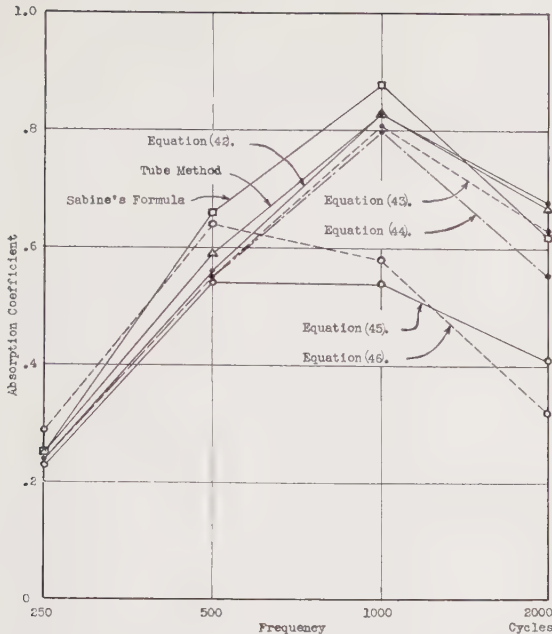


FIG. 11. Random incidence absorption coefficients for Acousti-Celotex.

density associated with the three components of particle velocity. In addition, energy density measurements were taken at ten selected points along two semi-diagonals of the room. From this data, absorption coefficients were computed using Eqs. (42), (43), (44), (45), and (46). The results are plotted in Figs. 10 and 11. The curves marked "Equation (42)," "Equation (43)," and "Equation (44)" were obtained from measurements of power density and energy density made only at the surface of the sample. The curve marked "Tube Method" was computed from the normal incidence coefficients as measured by the wattmeter for the material cemented in the tube. The conversion was made by first computing normal mobility, then using Eq. (50) giving the relation between normal mobility and random

incidence absorption coefficient. It is seen that these four curves agree fairly well for both types of absorbing material. The curves marked "Sabine's Formula" were obtained from reverberation time measurements made with the sample left in the same position. The agreement with the rest of the data is not quite as complete here. The curves marked "Equation (45)" and "Equation (46)" were computed from measurements of power density at the surface of the sample and energy density out in the body of the room as represented by the ten chosen points. Here the results are obviously unreliable. This may be due to the inability of the theory to cover the experimental conditions chosen, or to the use of too few observation points out in the body of the room.

A few applications may be suggested for the new power-measuring instrument described above. With its aid it should be possible to measure directly the sound power output of a loudspeaker or other electro-acoustic device. In noise reduction problems, the wattmeter may be of use in locating sound radiating surfaces. It has been demonstrated that in combination with the pressure and velocity units used separately, it can be used to measure acoustic mobilities by the "voltmeter-ammeter-wattmeter" method. A new method is also provided for measuring the absorption coefficient of a material either as a small sample mounted in a tube, or as a large sample on the wall of a room. In this connection, it promises to furnish the first means of measuring directly the individual absorption coefficients of different wall surfaces in an acoustically treated room.

The junior author takes this opportunity to thank Dr. P. H. Geiger and the Department of Engineering Research for the use of several valuable pieces of equipment.

